

## Optimal Liouville theorems for supersolutions of elliptic equations with the Laplacian

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**Abstract.** In this paper we consider the question of nonexistence of positive supersolutions of the equation  $-\Delta u = f(u)$  in exterior domains of  $\mathbb{R}^N$ , where  $f$  is continuous and positive in  $(0, +\infty)$ . When  $N \geq 3$ , we find that positive supersolutions exist if and only if

$$\int_0^\delta \frac{f(t)}{t^{\frac{2(N-1)}{N-2}}} dt < +\infty$$

for some  $\delta > 0$ . A similar condition is found for  $N = 2$ : positive supersolutions exist if and only if

$$\int_M^\infty e^{at} f(t) dt < +\infty$$

for some  $a, M > 0$ . The proofs are extended to consider some more general operators, which include the Laplacian with gradient terms, the  $p$ -Laplacian or uniformly elliptic fully nonlinear operators with radial symmetry, like the Pucci's extremal operators  $\mathcal{M}_{\lambda, \Lambda}^\pm$ , with  $\Lambda > \lambda > 0$ .

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